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A Graph Theoretic Approach to the Investigation of System-Environment Relationships[1]

The purpose of this paper is to undertake a graph theoretic analysis of those empirical structures for which it is meaningful to consider a part of the structure as a system and the remainder as its environment. We shall be primarily concerned with social systems such as groups, social networks or organizations, but the results are applicable to any structure that can be appropriately represented by means of graph theory.

The term *empirical structure* is here used to refer to a set of empirical elements (such as people, roles or positions) together with a set of empirical relationships (such as liking, communication or influence) between pairs of elements. Given such a structure, its digraph D is obtained as follows. The set of points of D, denoted $V = \{v_1, v_2, \ldots, v_p\}$, corresponds to the set of empirical elements, and there is an arc (directed line) from v_i to v_j if and only if the corresponding ordered pair of elements is in the specified empirical relation. Throughout the following discussion we shall be concerned with relationships that can be viewed as "links" of communication by which "messages" are transmitted from one element to another. In graph theory, a *walk* W of D is defined as an alternating sequence of points and arcs which begins and ends with a point and has the property that each arc is preceded by its first point and followed by its second one. Thus each walk of D represents a "chain of communication" within the structure, and it is possible for a message to reach element v_j from v_i if and only if D contains a walk from v_i to v_j.

It should be noted that the term *message* is used here to refer to anything that can be transmitted from one element to another. Thus, for example, if the elements of a structure are thought of as roles, subgroups or positions of an organization, then a message might be a person, memorandum, unit of work or some other "object" that can change its location in the organization. If, on the other hand, elements are taken to be individual people, then a message might be any of the following: (a) an item of information, an opinion or a rumor; (b) an influence attempt, such as an order, request or suggestion; (c) some ma-

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terial object that may be given, lent or sold; or (d) some symbolic object such as a favor, approval, help or support. This latter type of interpretation has been employed in research on social networks, as reported by Mitchell (1969) and Barnes (1972), where it is assumed that a variety of such messages can be transmitted through a given network. A link that carries more than one kind of message is said by these authors to be "multistranded."

Let us assume that we have some empirical basis for identifying a particular system within a given structure so that the set V of points of its associated digraph can be partitioned into two subsets, $S = \{s_1, s_2, \ldots, s_m\}$, corresponding to the elements of the system, and $E = \{e_1, e_2, \ldots, e_{p-m}\} = V - S$, corresponding to the elements of its environment. Strictly speaking, a *system* and its *environment* correspond to the subgraphs $\langle S \rangle$ and $\langle E \rangle$ that are induced by their sets of points,[2] but it will be convenient to denote them more simply by S and E.

For illustrative purposes, two rather different sorts of interpretation will be employed throughout this paper. In the first, we assume that a digraph D represents the *interpersonal communication structure* of a group of people and that system S is a specific subgroup within the group. Messages are taken to be items of information which can be transmitted only via links of the structure. Thus, points correspond to individual people, arcs correspond to links of interpersonal communication and walks indicate permissible chains of communication. In the second interpretation, we assume that D represents the *career structure* of an organization and that S represents a specified department within the organization. Here, points correspond to positions (or jobs), messages are thought of as individual people, arcs indicate permissible changes of position and walks designate permissible career lines of individuals.

Figure 1 shows the digraph of a small structure containing a system, $S = \{s_1, s_2, s_3\}$, and its environment, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$. It can readily be seen that in this digraph every walk W between a point of S and a point of E must contain arcs s_1e_1 or e_1s_1 and hence points s_1 and e_1. The corresponding communication links and elements of the structure are thus uniquely involved in all transactions between the system and its environment and would seem intuitively to be located in the "boundary" between S and E.

If this digraph is interpreted as an interpersonal communication structure, then all chains of communication between a member of subgroup S and a nonmember must contain "boundary" persons s_1 and e_1 who are uniquely able to monitor, modify or intercept any item of information going between the subgroup and its environment. It is also reasonable to assume, in keeping with the analysis of the functions of gossip in social networks presented by Epstein (1969), that when an item of information traverses a "boundary link" of such

[2]The definitions of concepts of graph theory that are not given in this paper may be found in Harary, Norman and Cartwright (1965).

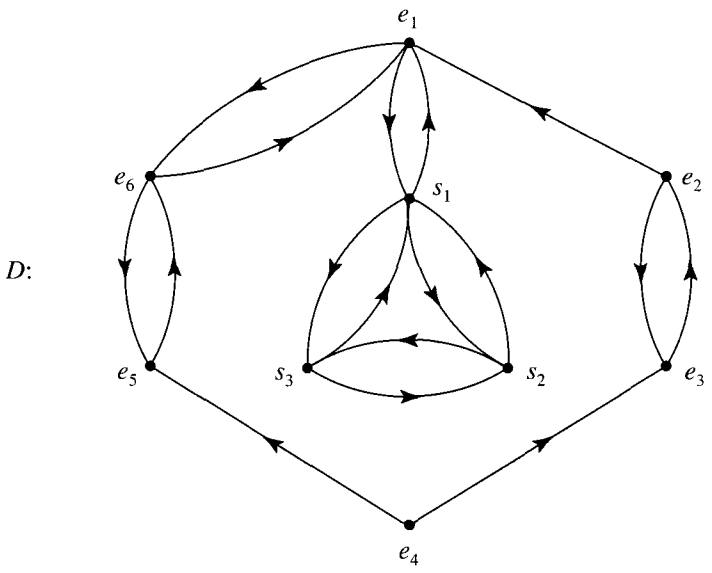

Figure 1. The digraph of a system $S = \{s_1, s_2, s_3\}$ and its environment $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$.

a system, its content or meaning may be significantly altered. In a similar way, if D is interpreted as the career structure of an organization, then all career lines that include positions in both department S and its environment must contain "boundary positions" s_1 and e_1. Anyone entering or leaving the department can do so only by going directly from one of these positions to the other, and when a person traverses a "boundary link" in the career structure, he changes departmental membership and consequently becomes subjected to a different set of influences and expectations.

The concept of boundary has been employed extensively by theorists (Emery, 1969; Katz and Kahn, 1966; Miller and Rice, 1967; Rice, 1963) who adopt a "systems approach" to social organizations. According to this view, those members of an organization who are directly involved in transactions between the organization and its environment are said to occupy "boundary roles" and various consequences have been identified for such individuals. Thus, for example, Kahn et al. (1964) have reported that occupants of boundary roles are more likely to experience role conflict than are members located "deep within the organization." The explanation proposed by Kahn et al. to account for this difference can be elucidated by means of the digraph shown in Figure 1. Let us assume that S and E represent a (miniature) organization and its environment, that the points of D correspond to individuals, that role expec-